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Forced convection in a porous medium heated by a permeable wall perpendicular to flow direction: analyses and measurements

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Abstract

The forced convection in a saturated porous medium subjected to heating with a permeable wall perpendicular to the flow direction is investigated analytically. It is shown that the heat transfer rate from the permeable wall to the fluid can be described by a simple equation: $Nu = Pe$. As compared with $Nu \propto Pe^{1/2}$ for the case of boundary lay flow over a flat plate embedded in a porous medium, the linear relationship between Nu and Pe indicates that heat transfer can be remarkably enhanced for the case when the fluid flow direction is opposite to the heat flow direction. To verify the analytical solution, experiments were carried out in a porous structure consisting of glass beads heated by a finned surface. The analytical solution is shown to be reasonable agreement with the experimental data.

Keywords: Porous media; Heat transfer; Heat transfer enhancement; Forced convection

1. Introduction

Convective heat transfer in a confined porous medium has been a subject of intensive studies during the past two decades because of its wide applications including geothermal energy engineering, groundwater pollution transport, nuclear waste disposal, chemical reactors engineering, insulation of buildings and pipes, and storage of grain and coal, and so on. Cheng [1] provides an extensive review of the literature on convection heat transfer in fluid saturated porous media with regard to applications in geothermal systems. The state of art concerning porous media models has been summarized in the book by Nield and Bejan [2] as well as the book by Kaviany [3]. For the case of boundary layer flow over a flat plate embedded in a porous medium, the overall Nusselt number over the plate heated at a constant heat flux is given by [2]

$$Nu = 1.329 Pe^{1/2}, \tag{1}$$

where the Nusselt number is defined as $Nu = (hd_p/k_e)$ and the Peclet number as $Pe = (d_p U/k_e/\rho c_p)$, with h being the heat transfer coefficient, d_p the diameter of the porous medium, U the velocity, and ρ, c_p, as well as k_e representing the fluid density, the fluid specific heat, and the effective thermal conductivity of the porous medium, respectively.

Recently, increased demands for dissipating high heat fluxes from electronic devices, high power lasers, and X-ray medical devices have created the need for new cooling technologies as well as improvements in existing technologies. To meet such demands, different cooling schemes have been proposed. One major category of heat exchangers for such applications is referred to as porous media heat exchangers [4]. The basic idea of the porous media heat exchangers is that enhanced cooling can be achieved because (i) larger surface areas available in porous particles as extended surfaces for heat transfer and (ii) mixing of fluids due to the presence of particles. A pumped single-phase porous media heat exchanger has recently demonstrated the capability for removing

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Notation

B	shape factor for a packed bed
c_p	specific heat (J/(kg K))
d_p	diameter of the solid particle, m
h	heat transfer coefficient ($W/(m^2K)$)
K	permeability (m^2)
k_e	effective thermal conductivity (W/(m K))
k_f	thermal conductivity of liquid (W/(m K))
k_s	thermal conductivity of solid (W/(m K))
Nu	Nusselt number
P	pressure (Pa)
Pe	Peclet number
Pr	Prandtl number
q	imposed heat flux (W/m^2)
Re	Reynolds number
T	temperature (K)
T_i	inlet temperature of fluid (K)
T_w	wall temperature of heating surface (K)
u	Darcian velocity (m/s)
U	velocity (m/s)
x	coordinate in horizontal direction
y	coordinate in vertical direction

Greek symbols

φ	porosity of porous medium
λ	k_f/k_s
μ	viscosity (Pa s)
ρ	density (kg/m^3)
θ	dimensionless temperature
θ_w	dimensionless temperature at wall surface
ξ	dimensionless distance

high heat fluxes [5]. One of the major disadvantage using a porous heat exchanger, however, is the large pressure drop across the heat exchanger. In order to overcome this disadvantage, one of the best methods is to reduce the flow velocity while keeping a higher heat transfer coefficient. This implies that the study of heat transfer enhancement in porous media is important. Vafai and Kim [6] investigated the thermal performance of a composite porous medium–fluid system, with the porosity and the effective thermal conductivity assumed to be constant, the enhancement of the thermal performance of the porous medium mainly depends on the ratio of the effective thermal conductivity of the porous medium to the fluid thermal conductivity. Heat transfer can be significantly enhanced when the ratio is sufficiently greater than the unity. Hadim [7] studied a fully porous channel and a partially divided porous channel. The results indicated that the partially divided porous channel configuration was an attractive heat transfer augmentation technique, although the ratio of the effective thermal conductivity of the porous medium to the fluid thermal conductivity was equal to 1. Huang and Vafai [8] conducted experiments and numerical methods to study the thermal performance of a porous channel, and the results illustrated that thermal enhancement could be obtained by using a high thermal conductivity porous medium. Fu et al. [9] numerically investigated the thermal performance of a porous block mounted on a partially heated wall in a laminar-flow channel. The results indicated that thermal performance was enhanced for the partially blocked situation by using a porous block with a higher porosity and bead diameter, however, the results were opposite to those of the fully blocked situation.

The present study was motivated by the work in heat transfer enhancement in a plain medium by Guo et al. [10,11] and the objective is to show that heat transfer can be enhanced by adjusting the included angle between the directions of the fluid flow and the heat flow. Guo et al. [10,11] analytically examined the energy equation for the boundary layer flow and showed that included angle between the fluid flow direction and the heat flow direction plays an important role on heat transfer enhancement in addition to increasing Reynolds number and the uniformity of velocity and temperature profiles. Moreover, they showed that the heat transfer rate reaches the maximum when the fluid flow direction is opposite to the heat flow direction. Here, we shall analytically examine the forced convection in a saturated porous medium subjected to heating with a permeable wall perpendicular to the flow direction. We shall show that under this situation, the Nusselt number is linearly increased with the increase of the Peclet number as compared with $Nu \propto Pe^{1/2}$ for the case of boundary lay flow over a flat plate embedded in a porous medium. Furthermore, in order to verify the analytical solution, we have also performed experiments in a porous structure consisting of glass beads heated by a finned surface. The analytical solution is shown to be reasonable agreement with the experimental data.

2. Analytical

The problem to be considered is schematically depicted in Fig. 1. A permeable plate is embedded in a semi-infinite porous medium. A fluid at infinity with a temperature T_i, flowing upwards through the porous medium, is heated by the downward-facing permeable plate with a constant heat flux q. To simplify the analysis, we assume that the porous medium is rigid, uniform, isotropic and fully saturated with fluid. It is

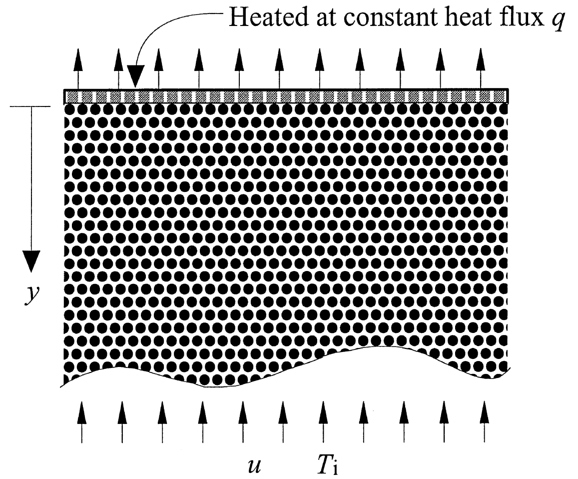

Fig. 1. Physical model and coordinate system.

further assumed that the thermophysical properties of both solid and fluid are constant, the fluid and solid phases are in local thermal equilibrium, and the thermal dispersion is negligible. With these assumptions, the governing equation for energy conservation is given by

$$-\frac{d(uT)}{dy} = \frac{d}{dy}\left(\frac{k_e}{\rho c_p}\frac{dT}{dy}\right), \tag{2}$$

where ρ and c_p denote the density and specific heat of the fluid, respectively. The effective thermal conductivity of the porous medium k_e is given by

$$k_e = k_f\left\{1 - \sqrt{1-\varphi} + 2\frac{\sqrt{1-\varphi}}{1-\lambda B} \times \left[\frac{(1-\lambda)B}{(1+\lambda B)^2}\ln\left(\frac{1}{\lambda B}\right) - \frac{B+1}{2} - \frac{B-1}{1-\lambda B}\right]\right\}, \tag{3}$$

where $\lambda = k_f/k_s$ with k_s and k_f being the thermal conductivity of solid and liquid phases, respectively; $B = 1.25((1-\varphi)/\varphi)^{10/9}$ represents the shape factor for a packed bed consisting of uniform spheres.

When a pressure gradient is given, the velocity u in Eq. (2) can be obtained based on Darcy's law:

$$u = -\frac{K}{\mu}\frac{\partial P}{\partial y}, \tag{4}$$

where the permeability K is given by

$$K = \frac{\varphi^3 d_p^2}{180(1-\varphi)^2}, \tag{5}$$

with d_p and φ representing the diameter of the spheres and the porosity of the porous medium, respectively.

Solving Eq. (2) subjected to the following boundary conditions:

$$q = -k_e\frac{dT}{dy} \quad \text{at } y = 0, \tag{6}$$

and

$$T = T_i \quad \text{at } y \to \infty \tag{7}$$

yields

$$T - T_i = \frac{q}{c_p\rho u}\exp\left(-\frac{c_p\rho u}{k_e}y\right). \tag{8}$$

Introducing the dimensionless quantities $\xi = y/d_p$ and $\theta = ((T - T_i)/qd_p/k_e)$, we can rewrite Eq. (8) as

$$\theta = \frac{\exp(-Pe\,\xi)}{Pe}, \tag{9}$$

where the Peclet number $Pe = RePr$, with the Reynolds number $Re = d_p\rho u/\mu$ and the Prandtl number $Pr = c_p\mu/k_e$. To evaluate the heat transfer rate from the heated permeable plate to the fluid, we define the Nusselt number Nu as

$$Nu = \frac{qd_p}{k_e(T_w - T_i)} = \frac{1}{\theta_w}, \tag{10}$$

where the temperature at the heated plate θ_w can be obtained from Eq. (9) by letting $\xi = 0$ to give

$$\theta_w = \frac{1}{RePr}. \tag{11}$$

Substituting Eq. (11) into Eq. (10), we obtain

$$Nu = Pe. \tag{12}$$

Eq. (12) indicates that the Nusselt number is increased linearly with the increase of the Peclet number for the case in which the flow direction is opposite to the heat flow direction for forced convection in a porous medium. On the other hand, for the case of boundary layer flow over a flat plate in which the included angle between the flow direction and the heat flow direction is 90°, Eq. (1) suggests that Nusselt number is proportional to the square root of the Peclet number. Therefore, we can conclude that heat transfer can be significantly enhanced by devising a heat transfer device such that that the included angle between the directions of the fluid flow and the heat flow is close to or equal to 0.

3. Experimental

In order to verify the above analyses, experiments were performed in the test section shown in Fig. 2. The vertically oriented test section, 31.5 mm in height, 99 mm in width, and 28 mm in depth, was enclosed with four vertical walls: three Teflon plates located in the both lateral and the back sides and one transparent Pyrex glass plate located in the front side. A perforated plate was installed at the bottom of the test section to

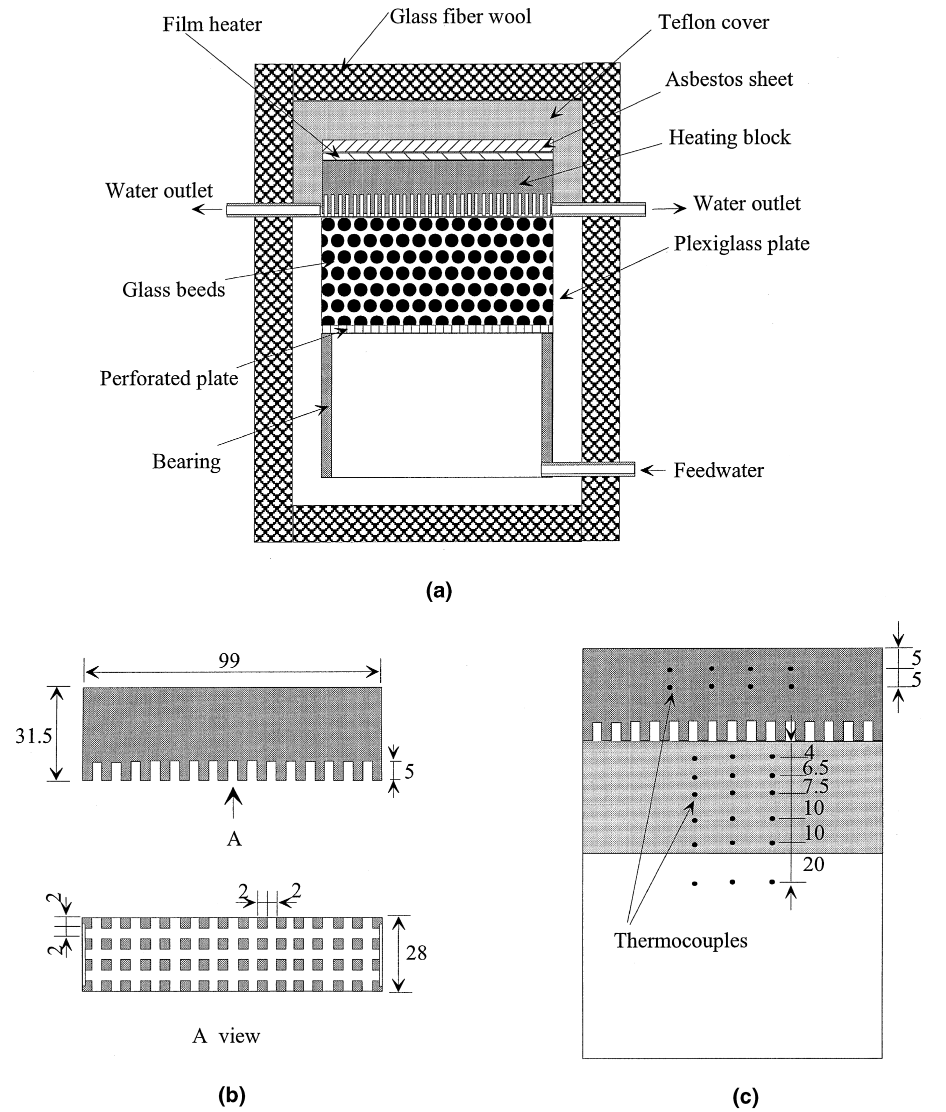

Fig. 2. Test section and locations of the thermocouples: (a) schematic of the test section; (b) heating copper block; (c) locations of thermocouple.

hold the glass beads in a desired place, and also, to act as a flow distributor. Spherical glass beads having an average particle diameter of $d_p = 1.09$ mm were packed into the test section.A finned copper block, shown in Fig. 2(b), was then carefully mounted on the top of the porous structure such that the copper block fins were in good contact with the porous medium. A heating assembly, consisting of a stainless steel film heater (0.1 mm in thickness), a mica sheet between the film heater and copper block serving as an electrical insulator, and an asbestos sheet with a Teflon cover plate serving as a heat insulator, was placed on the top of the copper block. The power supply of the film heater was provided by 220 V commercial power source through an ac automatic voltage regulator and a voltage-regulating transformer. All of the boundaries of the test section were insulated using glass fiber wool.

The experimental system was schematically illustrated in Fig. 3. The working fluid, deionized water, drained from a water tank, entered the test section from its bottom, flowed toward the finned heating block, and exited from the two outlet tubes located in the two sides of the test section. The inlet temperature of water were controlled by a RTD controller with a 1.5 kW electric heater located below the packed column. The flow rate, adjusted by a needle valve, was measured by weighing the accumulated water in a container adjacent to the test section using a digital scale for a period of time.

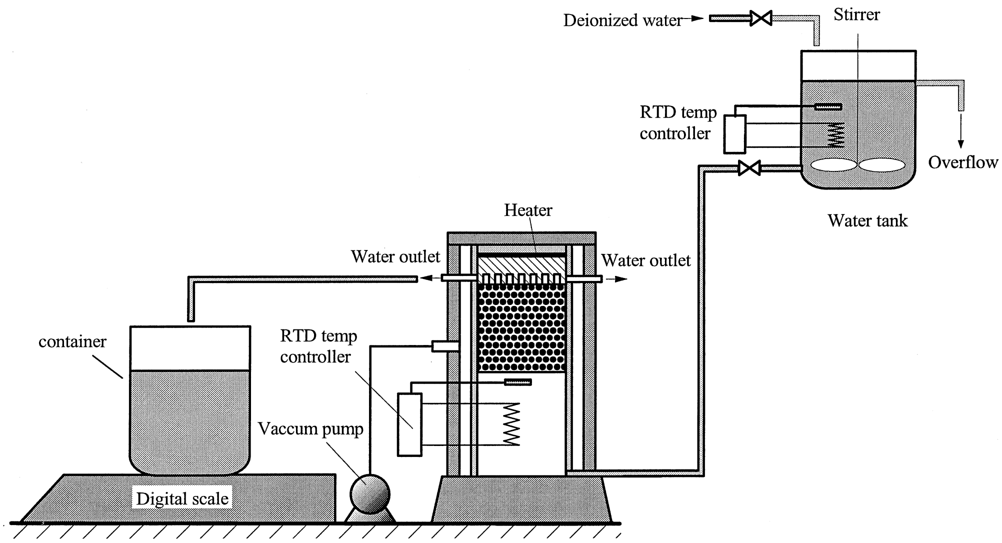

Fig. 3. Schematic of the experimental apparatus.

As sketched in Fig. 2(c), eight thermocouples were used to measure the temperature distributions in the heating copper block, whereas 15 were inserted in the porous medium to measure the temperature distributions in the porous medium. In addition, three thermocouples were used to monitor the temperature of the fluid entering the porous structure. The temperature at the fin tips of the heating block T_w was obtained by extrapolating the measured temperatures in the copper block to the surface with one-dimensional conduction heat transfer assumption. All the thermocouples used in the present study were T-type and had a diameter of 0.8 mm. A data acquisition system, consisting of a personal computer, an A/D converter board (MetraByte DAS-20), and two universal analog input multiplexers (MetraByte EXP-20), were employed to record the temperature measurements.

All the thermocouples were calibrated to ensure the accuracy within ±0.2°C. It was estimated that the uncertainty of the mass flow rate was ±0.4% while the uncertainty of the imposed heat load was about 9.0%, which was primarily caused by the heat loss. It was estimated that the uncertainty of the heat transfer coefficient was about 12.0% with the uncertainty estimation method proposed by Kline and McClintock [12].

4. Results and discussion

4.1. Temperature distributions

The analytical solution, Eq. (9), is presented in Fig. 4 for various Peclet numbers. It can be seen that for a given Peclet number the temperature in the porous medium progressively decreases away from the heated permeable wall. It is also clear from this figure that the increase of the Peclet number leads to a decrease in the wall temperature. Moreover, the temperature gradient in the porous medium seems somewhat steeper as the Peclet number is increased. These facts imply that the heat transfer from the wall to the bulk fluid is enhanced with the increase of the Peclet number, as indicated by Eq. (10).

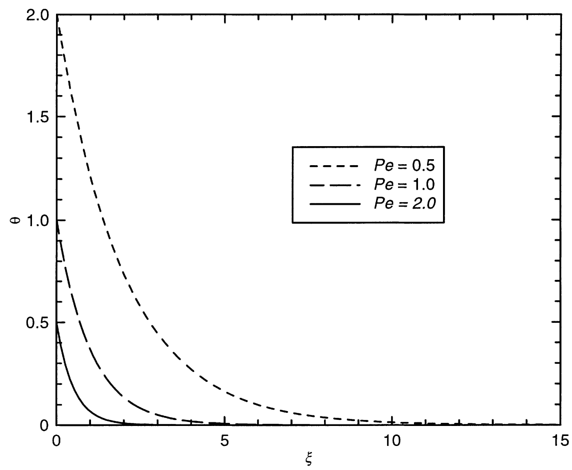

Fig. 4. Temperature distributions for various Peclet numbers.

The analytical solution, Eq. (8), and the measured temperature distributions in the porous medium are compared in Figs. 5 and 6 for various heat fluxes at the inlet temperature of 30°C. It is seen that from Fig. 5,

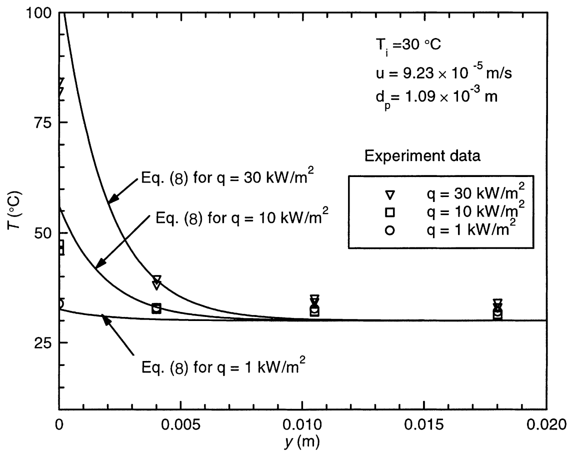

Fig. 5. Comparison between the predicted and the measured temperatures at low velocity.

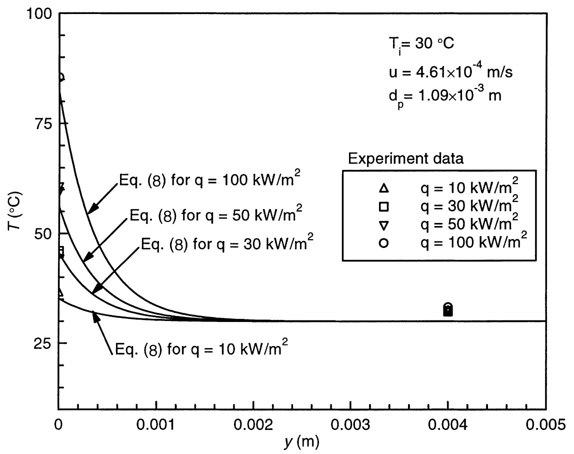

Fig. 6. Comparison between the predicted and the measured temperatures for a high velocity.

that for a small flow velocity of $u = 9.23 \times 10^{-5}$ m/s, the analytical solution is generally in reasonable agreement with the experimental data with certain deviations at the wall. Fig. 6 indicates that as the flow velocity is increased to $u = 4.61 \times 10^{-4}$ m/s, the agreement between the theoretical and the experimental results is improved.

The experimental data for the Nusselt number against the Peclet number are presented in Fig. 7. For comparison, the analytical solution, Eq. (12), is also plotted in the same figure. The symbols represent the measured data, whereas the solid line represents the analytical solution. It is shown that the analytical solution is in good agreement with the experiment in the range of small Peclet numbers. However, it is evident from Fig. 7 that the analytical solution, Eq. (12), deviates from the measured data for higher Peclet numbers.

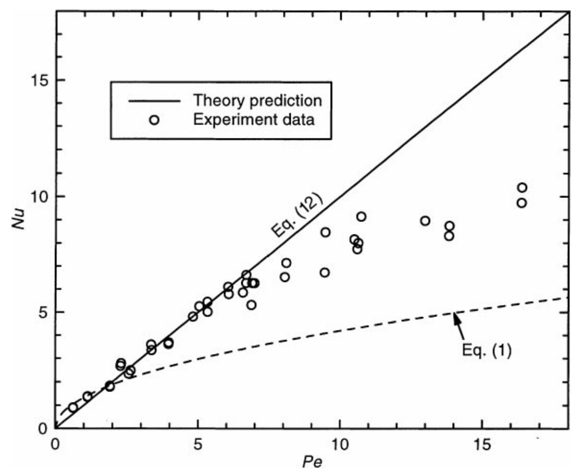

Fig. 7. Comparison between the predicted and the measured Nusselt numbers.

The discrepancy between the analytical solution and the experimental data for higher Peclet numbers may be mainly attributed to the fact that the theoretical problem under consideration is slightly different from the problem investigated experimentally. Theoretically, we consider a problem of forced convection in a porous medium, which is subjected to heating by a permeable wall perpendicular to the flow stream direction. It is assumed that the included angle between the velocity and the temperature gradient is zero such that an one-dimensional approximation can be applied to analyze the problem. By doing so, our motivation is to theoretically demonstrate that the heat transfer can be significantly enhanced with the present configuration as compared with other cases in which the flow velocity is not parallel with the temperature gradient such as the case of the boundary layer flow over a flat plate embedded in a porous medium. The problem considered theoretically may be investigated experimental by construct a porous structure heated by a perforated plate such that the working fluid can flow through the porous medium and exit directly from the heated perforated plate. At low mass flow rates, the flow field in the experimental setup is quite close to the theoretical model. However, at high flow velocities, the flow field in the experimental setup may become different from the theoretical model. As a result, the flow velocity in the vicinity of the fins is not normal to the heating surface. However, a comparison between the experimental data from the present setup with the case of the boundary layer flow over a flat plate embedded in a porous medium, it is evident from Fig. 7 that the present configuration exhibit a substantially higher heat transfer rates, suggesting that the present configuration can be used to design a heat device of higher heat transfer rates.

5. Concluding remarks

Forced convection in a saturated porous medium subjected to heating with a permeable wall perpendicular to the flow direction has been examined analytically. The heat transfer rate from the permeable wall to the bulk fluid for such a heat transfer configuration has been shown to be described by a simple equation: $Nu = Pe$. The comparison between this linear equation with $Nu = Pe^{1/2}$ for the case of boundary lay flow over a flat plate embedded in a porous medium suggests that heat transfer can be significantly enhanced when the flow direction is parallel to the applied temperature gradient. The experiments performed in a porous structure consisting of glass beads heated by a finned surface have shown the analytical solution in good agreement with the experimental data at low Peclet numbers.

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